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ASSESSMENT OF THE CRITICAL STATE OF THE POLYETHYLENE PIPE IN THE AREA OF THE MECHANICAL METAL–POLYMER JOINT AT LOW CLIMATIC TEMPERATURES

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This paper analyzes the distributions of shearing forces arising in the area of the mechanical joint of polyethylene and metal pipes. The value of these forces at low climatic temperatures (down to –60^{o}C) is determined. The results of the theoretical analysis for isothermal conditions permit estimating the working capacity of plastic under different conditions of joints with metal pipes.

In metal pipelines, to neutralize the action of temperature and mechanical deformations, expansion U-bends are used, as a rule. For polymer pipes at $R/h > 16$, the problem of axial displacement compensation becomes more acute. It can be solved by the engineering-design method with the use of different combinations of materials. The point of the method is the limitation of the possible radial displacements of the polymer pipe at joints at a relative freedom of axial and tangential displacements.

Bending of the structure and axial compression or tension in the presence of joints increase, in the final analysis, the tangential forces, which become, in general, self-balanced due to the structure symmetry, and under normal temperatures they rapidly decay due to the damping effect of the polymer material. Nevertheless, even at high temperatures, as practice shows, these forces lead to a premature destruction of polymer shells in the near-boundary region of the mechanical joint. It is not known what will happen if the shell is homogeneous in thickness and has a large ratio R/h. Apparently, the exact solution of this problem in a three-dimensional formulation requires rather serious theoretical study. In the present paper, by solving the plane problem, an attempt is made to answer the following questions: how deep into the polymer shell does the influence of shearing forces go and what is their value at low climatic temperatures.

The cylindrical shell under consideration is long enough so that the mutual influence of the ends is insignificant. If the shell is sealed at the ends by the joint, the fixing conditions can be of three types: free, semirigid, and rigid. The "free" condition is modeled by the compensator in the form of cylindrical systems with parallel-series elastic elements, the "semirigid" condition – in the case of using a design with a damping set of heterogeneous materials, and the "rigid" one — in the case of an immobile metal-polymer connection. It was assumed that the thermoplastic layer was rigid and the Kirchhoff–Love hypotheses were applicable to it. The physical part of the problem — the influence of thermoelastic stresses in the semi-infinite shell — was replaced by specified forces and the account of the temperature changes in the material — by elastic constants. Thus, the elastic constants of the material, not being functions of the temperature, have different values at different temperatures. Actually, to solve the problem, their dependence was determined from experimental data. We used the familiar relations of the theory of cylindrical shells and the corresponding solution of the boundary-value problem with the boundary conditions [1–4].

The X-axis is directed along the cylinder axis and the Y-axis — along the radius. Going to two independent coordinates $\theta = Y/r$ and $\alpha = X/r$, we carry out the solution on the basis of the theory of gently sloping cylindrical shells [1] describing the stressed-strained state of the pipe:

$$\nabla^2\nabla^2\nabla^2\nabla^2 W + D^* \frac{\partial^4 W(\alpha, \theta)}{\partial \alpha^4} = 0 , \tag{1}$$

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where ∇^2 is the Laplace operator and $D^* = 12(1-\nu^2)\, r^2/h^2$.

Determine the forces and the deflection:

$$N_1 = Eh_r \frac{\partial^4 W}{\partial\alpha^2 \partial\theta^2}, \quad \tau_1 = -Eh_r \frac{\partial^4 W}{\partial\alpha^3 \partial\theta}, \quad N_2 = Eh_r \frac{\partial^4 W}{\partial\alpha^4}, \quad M_1 = \frac{D}{r^2}\left(\frac{\partial^2}{\partial\alpha^2} + \nu \frac{\partial^2}{\partial\theta^2}\right)\nabla^2\nabla^2 W,$$

$$M_2 = \frac{D}{r^2}\left(\frac{\partial^2}{\partial\theta^2} + \nu \frac{\partial^2}{\partial\alpha^2}\right)\nabla^2\nabla^2 W, \quad Q_1 = \frac{D}{r^3}\left[\frac{\partial^3}{\partial\alpha^3} + (2-\nu)\frac{\partial^3}{\partial\alpha\partial\theta^2}\right]\nabla^2\nabla^2 W, \tag{2}$$

$$\omega = \nabla^2\nabla^2 W, \quad D = \frac{Eh^3}{12\,(1-\nu^2)}, \quad h_r = \frac{h}{r},$$

where E is the Young modulus of the material ($E = E(T)$ and $\nu = \nu(T)$ are experimental values). We seek the solution of Eq. (1) in the form

$$W = W_1 \exp(k\alpha) \sin n\theta, \tag{3}$$

since the influence of the opposite ends is ignored, W_1 and k are the coefficients to be determined, and n is a real integral number ($n > 2$).

We replace the various external forces and the stresses arising from the physical action on the shell by a system of external forces S_J (J is the number of forces) applied to the edge contour from the side of the shell. Denote the reaction of the resultant of these forces with respect to the fixed contour by $\tau = S_0^T \sum\limits_J |S_J|$. It will be shearing and self-balanced on the circular section contour. The value of S_0^T was determined from the experimental data and calculated by the empirical formula $S_0^T = 0.2E(T)\cdot 10^{-3}$. We give the law of change in the shearing force per unit length on the θ-coordinate by the relation $\tau = \tau_0 \cos(n\theta)$.

The general integral of Eq. (1) with allowance for the loading symmetry and its eight pairwise conjugate roots of the characteristic equation will be of the form

$$W(\alpha, \theta) = [\exp(-\lambda_1\alpha)(c_4 \cos\lambda_2\alpha + c_8 \sin\lambda_3\alpha) + \exp(-\lambda_3\alpha)(c_2 \cos\lambda_4\alpha + c_6 \sin\lambda_2\alpha)] \sin n\theta, \tag{4}$$

where c_2, ..., c_8 are arbitrary constants (constants with odd subscripts are equal to zero);

$$\lambda_1 = \frac{\sqrt{2}}{4}(n_2 - \sqrt{b}\,); \quad \lambda_2 = \frac{\sqrt{2}}{4}(n_3 - \sqrt{b}\,); \quad \lambda_3 = \frac{\sqrt{2}}{4}(n_2 + \sqrt{b}\,); \quad \lambda_4 = \frac{\sqrt{2}}{4}(n_3 + \sqrt{b}\,);$$

$$n_1 = \sqrt{16n^4 + b^2}; \quad n_2 = \sqrt{4n^2 + n_1}; \quad n_3 = \sqrt{n_1 - 4n^2}; \quad b = \sqrt{D^*}.$$

Thus, the problem is actually reduced to the consideration of the semi-infinite shell. Substituting expression (4) into (2), we obtain the values of the internal forces and displacements

$$N_1^* = N_1\tau_0^{-1}, \quad N_2^* = N_2\tau_0^{-1}, \quad \tau_1^* = \tau_1\tau_0^{-1}, \quad Q_1^* = Q\tau_0^{-1}, \quad \tau^* = \tau\tau_0^{-1}, \quad M_1^* = M_1(\tau_0 r)^{-1}, \quad M_2^* = M_2(\tau_0 r)^{-1},$$

$$\omega^* = \omega E\tau_0^{-1}, \quad \frac{\partial\omega^*}{\partial\alpha} = \frac{\partial\omega}{\partial\alpha} E\tau_0^{-1}; \tag{5}$$

$$c_2^* = c_2 L_h, \quad c_4^* = c_4 L_h, \quad c_6^* = c_6 L_h, \quad c_8^* = c_8 L_h, \tag{6}$$

where $L_h = nEh_r/\tau_0$.

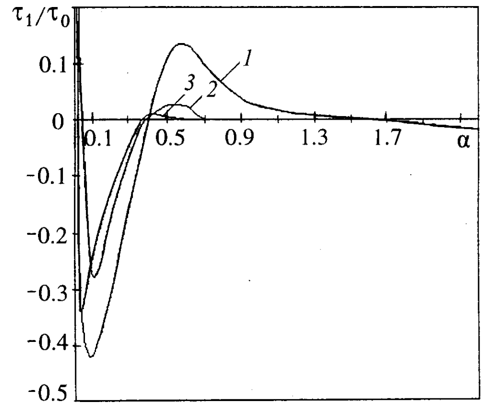

Fig. 1. Distribution of forces τ_1 over the polyethylene pipe axis in the case of "semirigid" fixing (T = 20°C, h_r^{-1} = 100) at the following values of n: 1) 4; 2) 8; 3) 16.

Here we omit the writing of the forces, momenta, displacements (N_1, N_2, τ_1, Q, M_1, M_2, ω, $\partial\omega/\partial\alpha$), and arbitrary integration constants (c_2, c_4, c_6, c_8) that are obtained analytically [3] because of their awkwardness.

Then, following the method of separation of variables, we determine the intermediate coefficients and find the arbitrary coefficients from the conditions at the edge at $\alpha = 0$:

a) free edge

$$N_1^* = 0\,,\quad \tau_1^* = \tau^*\,,\quad M_1^* = Q_1 = 0\,; \tag{7}$$

b) semirigid connection

$$N_1^* = 0\,,\quad \tau_1^* = \tau^*\,,\quad M_1^* = \omega^* = 0\,; \tag{8}$$

c) rigid connection

$$N_1^* = 0\,,\quad \tau_1^* = \tau^*\,,\quad \frac{\partial\omega^*}{\partial\alpha} = \omega^* = 0\,. \tag{9}$$

Conditions (7)–(9), taking into account expressions (5) and (6), permit writing the system of equations for determining the arbitrary coefficients c_i (i = 2, 4, 6, 8).

Thus, in terms of expressions (5) and (6) the internal forces and displacements in any section of the shell can be determined. They show that the character of change in the force factors and displacements is greatly influenced by the boundary conditions and the parameters n and h_r.

If we consider the shell to be free from external and internal forces on the surface, we arrive at the problem on the stability of the cylindrical shell compressed in the axial direction [4]. In so doing, it is assumed that the shell beyond the elastic limits is deformed to form local yielding regions beyond whose boundaries a combination of conditions providing the absence of shearing forces, i.e., the possibility of displacements along θ, is formed.

Consider our conditions. The mechanical arrangement of the shell edges restricts the possibility of their displacements. Displacements over the outer and inner surfaces of pipes are also restricted. The shearing forces arising at the shell edges can lead to an increase in the stresses and to the attaining of critical states.

Using dependences (5) and the approximating curves $E(T) = -0.0694T^2 - 23.65T + 1277.5$ and $\nu(T) = 10^{-0.5}T^2 - 0.0034T + 0.3904$, obtained by describing the experimental values of the elastic constants, we performed a computing experiment.

Before proceeding to the specific analysis of the shearing forces, we investigate the parameters n and h having a significant effect on the computational results. An increase or decrease in the parameter n can lead to unexpected results. For example, at n = 4 (T = +20°C, $h_r^{-1} = 100$) we have a force distribution inconsistent with reality (Fig. 1, curve 1) and the process of τ_1 decay along α has an asymptote other than $\tau_1\tau_0^{-1}$ = 0, which should not take place.

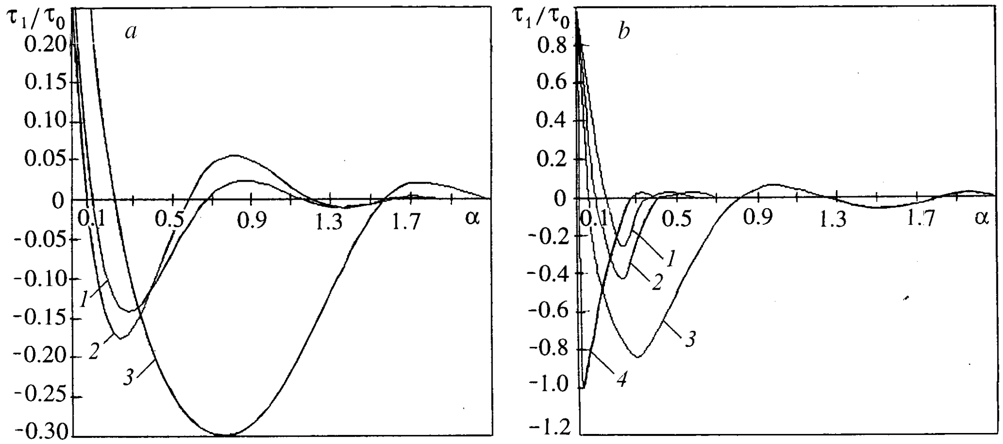

Fig. 2. Distribution of forces τ_1 over the polyethylene pipe axis at a temperature of 20^{o}C (a) and –60^{o}C (b) under the following fixing conditions: 1) "free"; 2) "semirigid"; 3, 4) "rigid" (curves 1–3 have been calculated for h_r^{-1} = 250; curve 4 — for h_r^{-1} = 500).

And for n = 16 (Fig. 1, curve 3) the decay process sharply enhances, which points to the inconsistency with the physical side of the problem. When n = 8, the process is the most real one (Fig. 1, curve 2).

Consider the parameter h_r that has a concrete physical meaning. A decrease in h_r, generally speaking, leads to an increase in the force τ_1 decay zone and, on the other hand, causes the negative maximum to move closer to the edge of the shell. Such a tendency is observed for all service temperatures (from +20^o to –60^{o}C) and fixing conditions, which is logical and complying with the physical side of the problem.

The most characteristic curves of the distribution of forces τ_1 along α at normal temperature (+20^{o}C) are given in Fig. 2a. It is seen that, as would be expected, the force τ_1 decays along the α-coordinate, periodically changing the sign independent of the fixing condition. In the case of "free" and "semirigid" fixing, the τ_1 decay zone is virtually the same. And at "rigid" fixing the decay zone is situated somewhat farther from the edges, with the forces being maximum.

If the temperatures are low (T = –60^{o}C, n = 8, h_r^{-1} = 250), the τ_1 distribution under different fixing conditions has the form of similar curves (Fig. 2b). Restraint of the radial displacements leads to an increase in the value of the shearing force, which in the zone of α = 0.3–0.4 approaches the critical value of $\tau_1\tau_0^{-1}$ = 1. The decay zone thereby reaches significant values ($\alpha = 2$), whereas in the cases with "free" and "semirigid" kinds of connections the decay zone is about four times smaller.

In the case of the critical values of the parameters in the model, i.e., at T = –60^{o}C, n = 8, and h_r^{-1} = 500 (Fig. 2b, curve 4), breakage of the thermoplastic is possible. This variant can be realized in the absence of internal pressure, i.e., when the pipeline is inoperative or at the moment it is shut off.

Thus, with decreasing service temperature, from the viewpoint of the accepted model, there is, naturally, an increase in the elastic modulus of the polymer shell and the computing experiment shows the appearance of dangerous zones of thermoplastic breakage. In this connection, it should be noted that in using the polyethylene pipeline, it is necessary to provide a strengthening band in the near-boundary regions of joints. This band can be made by applying wraps of thermoplastic in the form of a truncated cone.

In conclusion, note that the results obtained present only one attempt to consider the rather complicated practical problem by solving a homogeneous boundary-value problem. Here it is obvious that the damping properties of the polymer material have not been completely taken into account; they enter only through the corresponding coefficient and the actual data of the Young modulus of the material.

In general, the considered model makes it possible to analyze the possible variants in designing pipeline systems and gives new information for practical purposes by playing through critical situations.

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NOTATION

R, outer radius; h, pipe thickness; r, inner radius; X and Y, Cartesian coordinates; θ and α, dimensionless coordinates; $W = W(\alpha, \theta)$, potential function; ν, Poisson coefficient; D, flexural rigidity; N_1 and N_2, normal forces; τ_1, shearing force; T, temperature; θ_1, generalized cutting-off force directed along the cylinder normal; Q_1^*, dimensionless quantity of Q_1 (all dimensionless quantities are indicated by asterisks); M_1 and M_2, moments; ω, deflection; n, frequency of force τ_1; τ_0, amplitude of force τ_1; S_0^T, damping coefficient of the material depending on the temperature T, λ_1, λ_2, λ_3, λ_4, roots of the characteristic equation; h_r, geometric parameter of the pipe. Subscripts: r, dependence of the parameter on the inner radius of the polymer shell; h, dependence of the coefficient on geometrical quantities.

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